ANALISI DI STABILITA’ DI VEICOLI INCLINABILI A TRE RUOTE: CONFRONTO CON MOTOVEICOLI CONVENZIONALI

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Sommario
La necessità di muoversi in sicurezza nel traffico sempre più congestionato ha motivato lo sviluppo di veicoli stretti inclinabili a tre ruote che, coniugando l’agilità tipica dei motoveicoli e la sicurezza passiva degli autoveicoli, rappresentano una valida soluzione alla mobilità urbana convenzionale.

Il comportamento dinamico dei veicoli inclinabili a più di due ruote è per molti aspetti simile a quello dei motoveicoli. Esso è caratterizzato dai moti oscillatori di weave e wobble che devono essere investigati per prevedere la stabilità che inevitabilmente diminuisce all’aumentare della velocità.

Il presente lavoro si propone di studiare la stabilità di un veicolo inclinabile con assaleanteriore a due ruote e assale posteriore a ruota singola. I risultati sono confrontati con quelli di uno scooter della stessa categoria. Si sono evidenziati gli effetti di un avantreno a due ruote, di una differente configurazione dell’avantreno stesso oltre che dello smorzamento del canotto di sterzo e della rigidezza strutturale del telaio.

Abstract
The need of being able to travel safely and quickly in city traffic has motivated the development of narrow leaning vehicles, these vehicles can join the handling capability typical of motorbikes with the passive safety of cars and represent a suitable solution for conventional urban mobility.

Both dynamics of motorbikes and leaning vehicles are similar. These vehicles are characterized by oscillating weave and wobble motions that have to be investigated to study the vehicle stability which decreases with speed.

The aim of the present work is to study the stability of a leaning vehicle with two front wheels and one single rear wheel. The results are compared with those for a two wheels scooter of the same category. The results for a two wheels frontend are highlighted, together with those for a different frontend configuration, for variations of the steering bearing damping and of the frame structural stiffness.
Introduction

The problem of the mobility in urban and suburban areas is becoming a preminent matter in all the industrialized societies. The public transportation systems are efficient but are not able to solve the problem of individual mobility. Microcars seem to be not a valid solution especially in congested traffic. Motorcycles and scooters instead are much more efficient for mobility in the traffic of our cities. Nevertheless, the safety remains the open problem of such a transportation system. This is the reason why the interest in the development of three or four wheels tilting vehicles is increasing in the last years. The potentiality to join the requirements of high mobility and safeness is becoming topical.

There were several attempt of developing lean vehicles since the beginning of the ‘950 for about 20 years ([5]). Their failure was mainly related to the lack of an available technology. In last decade, the congestion of urban traffic, the problem of the pollution, the increment of the energy costs and the progress of the technology motivated a renewed interest in small and narrow vehicles for individual mobility. New concepts were proposed and new configurations were designed ([23]). Two of them, the Carver (Van der Brink) and the Piaggio Mp3 are present on the market now. The first one has mainly the characteristics and the performance of a sport microcar. It is realized with two rear non tilting wheels and a main frame that can roll actively. The Piaggio Mp3 is a maxiscooter having two front wheels. The suspension kinematics allows to the front wheels a camber angle equal to the body lean angle which is controlled by the driver. The front wheels track is contained in the lateral size of the scooter. This allows exploiting the advantages of the two wheels on the front axle without compromising the mobility in congested urban areas. Some of these advantages can be appreciated driving the vehicle and are based on subjective analysis. Others, such as the lateral stability, can be explained only with the support of mathematical models.

The aim of the present work is to investigate the lateral stability of the three tilting wheels scooter. The results are intended to be compared to that of an equivalent two wheels vehicle circulating from several years.

A multibody modelling approach ([4], [11], [12], [13]) will be undertaken to take into account the main dynamic effects outlined in the recent literature on motorcycles ([8] - [14], [10], [21]). The literature on motorbike’s dynamic is considered the most appropriate reference being that on commuters very poor. Only analytical first approximation models are available to illustrate specific control properties ([16], [17], [22]). The most sophisticated model is based on approximations that neglect relevant effects (i.e. chassis compliance, dynamic behaviour of the tires, suspension’s kinematics) ([15]). A multi-body model will therefore be developed using a multipurpose environment. Commercially available tools are not appropriate for non standard three or four wheels tilting commuters. The derivation of the equations of motion starting from an energetic approach is almost impossible considering the complexity of the problem.

Both the two wheels and the three wheels models were validated experimentally by step steer tests: an impulsive force applied to the handlebar while straight running.

Vehicle description

The layout of the vehicles studied in the present paper is reported (Figure 1). The two wheels motorcycle is a conventional scooter. The main body and the rear suspension of the three wheels prototype have the same characteristics of the first one while the front body is completely different.

The kinematics of the front suspension is designed to allow to the front wheels a camber angle equal to the body inclination (up to 40 degrees). The aim is to exploit the tires
camber thrust and reduce their side slip. The suspension is realised by two separate struts (1 in Fig. 1b) that support each of the front wheels. The vertical movement of each wheel is obtained by a trailing arm operating in longitudinal direction and connected to the strut by a spring damper element. The tilting degree of freedom is obtained by supporting the two struts with two parallel rigid bars (3) free to rotate about the front body. The joints linking the struts to the bars allow realizing a kinematic system similar to that of a double wishbone suspension and allow the steering degree of freedom. This is obtained by means of bearings aligned with $z_f$ axis of Figure 1. The steering mechanism is based on a handlebar connected to a steering column that ends with a lever. The lever is connected by a steering link to the struts. The steering mechanism has been designed with the objective of decoupling the tilting and suspension movements from the steering. The vehicle roll degree of freedom is free as in a motorcycle and the driver must stabilize the capsize mode by acting on the steering system.

![Figure 1: Reference frames used for modelling the two and three wheels scooters.](image)

**Model description**

The main objective of the dynamic model is the analysis of the stability in straight running. A model implementing the following features is required:

- kinematics and inertia of the steering line and of the front and rear suspension
- gyroscopy of the wheels
- chassis compliance ([10], [12], [19], [21])
- adequate tyre force model
- tyres relaxation length ([12], [13])

However, a set of assumptions has been introduced to limit the model complexity without compromising its effectiveness:

- the powertrain has not been modelled and the velocity $V$ has been considered as an input to the model ([20])
- the effects of the tyre curvature has not been considered ([12],[13], [18]-[20])
- the tyre lateral forces have been taken into account neglecting the lateral – longitudinal force combination and the side slip force camber thrust interaction
- the static friction in the steering line and in the suspension joints is modelled as a simple viscous damping

The above listed assumptions comply with the objectives of the present analysis to investigate the stability in the small during straight running at constant speed.

The vehicles models have been developed in Matlab/SimMechanics environment. This allows modelling all the vehicles parts in multi-body blocks and the tyres – ground interaction using Matlab models. Additionally, the automatic conversion from 3D CAD drawings to multi-body language is considered an outstanding support for the present
application. The CAD assemblies of Fig. 1 are therefore translated automatically in the multi-body models of Fig. 2 and Fig. 3. Such models are then completed introducing the tyres models and some additional parts such as the suspension shock absorbers which are modelled as lumped spring-damper elements.

Figure 2 shows the multi-body model for the two wheels scooter. The front wheel block (6) includes the mechanical properties of the wheel and the tyre-road contact force model that will be described in the second part of the present section.

In the front fork block (7) the front spring – shock absorber is modelled by a lumped spring–damper element.

The chassis body (central frame (5)) is then modelled as a rigid part, except for the lumped compliant element (8) that accounts for the steering column and frame stiffness. The frame compliance is modelled with a 2 d.o.f. spring-damper involving the lateral motion and the rotation about a longitudinal axis. The driver body is rigidly connected to the chassis. The effects of its deviation from the longitudinal plane of symmetry are neglected being them not relevant for the present stability analysis. The rear fork (9) has then been connected to the rear frame using a spring–damper block. The rear wheel (4) is then modelled as the front one.

The three wheels model architecture (Figure 3:) is the same of the two wheels one for what is concerned the rear body, the trailing arm of the rear suspension, the non suspended mass and the chassis compliance. The front body is completely different. It includes the struts and the tilting mechanism (block 3). The steering system block introduces to linkage of the front wheels to the steering column and then to the handle bar. It is worth to note that the mass and the inertia of each mechanical part are properly taken into account. The right part of Fig. 3 evidenced how the front non suspended mass and the front wheel block in Fig 2 splits.

Figure 2 :Two wheels scooter Matlab/SimMechanics model
The model implemented for computing the tyre–ground contact force is each wheel is described here below. The side slip angle ($\alpha$) is computed starting from analysing the rotation of the wheel with respect to the inertial reference frame XYZ and from the speed ($V$) of the wheel/ground contact point measured in the same frame. The wheel’s rotation is given by the rotation matrix $R$ supplied by Simmechanics wheel’s hubs block. The rows of $R$ matrix define, with reference to the inertial reference frame XYZ, a triple of vectors $n_{hx}$, $n_{hy}$ and $n_{hz}$ (Fig. 4)

\[
\begin{align*}
  n_{hx} &= \begin{bmatrix} R_{xx} \\ R_{xy} \\ R_{xz} \end{bmatrix} \\
  n_{hy} &= \begin{bmatrix} R_{yx} \\ R_{yy} \\ R_{yz} \end{bmatrix} \\
  n_{hz} &= \begin{bmatrix} R_{zx} \\ R_{zy} \\ R_{zz} \end{bmatrix}
\end{align*}
\]

that are fixed to the wheel hub. The origin of the triple is in the ground-wheel contact point, the $n_{hz}$ vector passes through the centre of the wheel, $n_{hx}$ lies in the middle wheel’s plane. The projection of such vectors on the ground plane (or ground reference frame) can be expressed as

\[
\begin{align*}
  n_x &= \begin{bmatrix} R_{xx} \\ \sqrt{R_{xx}^2 + R_{xy}^2} \\ \sqrt{R_{xx}^2 + R_{xy}^2} \end{bmatrix}, \\
  n_y &= \begin{bmatrix} R_{yx} \\ \sqrt{R_{yx}^2 + R_{yx}^2} \\ \sqrt{R_{yx}^2 + R_{yx}^2} \end{bmatrix}, \\
  n_z &= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
\end{align*}
\]

The components of $V$ in that frame are defined as

\[
\begin{align*}
  u &= V \cdot n_x, \\
  v &= V \cdot n_y.
\end{align*}
\]

From eq. 3 follows the computation of the side slip angle ($\alpha$)

\[
\alpha = \arctan\left(\frac{v}{u}\right).
\]
Being the force generated by the side slip angle $\alpha$ related to the distortion of the tyre carcass ([19]), the time or rolled distance delay is taken into account using a first order filter ([9], [12]):

$$\alpha = \frac{1}{s + 1} \alpha,$$  \hspace{1cm} (5)

where $V$ is the longitudinal speed of the vehicle, $s$ the Laplace variable and $\bar{\sigma}$ is given by the following expression depending on the wheel speed and $V$ and on the tyre cornering stiffness $K_{y}a$:

$$\bar{\sigma} = K_{y\alpha} \left( 9.694e^{-6} + 1.333e^{-8} + 1.898e^{-9}V \right).$$  \hspace{1cm} (6)

The numerical values reported in equation (6) are taken from ([12]).

The camber angle ($\gamma$) (Figure 4 b) is estimated following the same approach. The lateral force generated by the side slip angle ($\bar{\alpha}$) and the camber angle ($\gamma$) are computed implementing a look up table of the experimental curves reported in Figure 5. The effect of the longitudinal tyre force as well as the interaction of the forces generated by the side slip and camber angles are neglected as the present work is limited to a linearized analysis. The tyre–ground contact area has been assumed to be always on the symmetry plane of the wheel. This means that the effect of the transversal tire curvature, as widely discussed in ([1], [2], [3], [12], [13]), is not taken into account being this contribution non relevant on the straight running stability analysis.

The constraints of the wheels to the ground are considered as holonomous for simplicity (the overturning stability can be monitored analyzing the tyre – ground vertical forces). The control action of the driver is not taken into account in the present model. This means that the capsize mode remains unstable during the stability analysis. The low coupling between capsize and the other modes justifies such an assumption.
A set of experimental tests were carried out to validate the numerical models presented in the previous section. The scooters were instrumented with three gyroscopes, two double axis accelerometers and one wire potentiometer. Such sensors allow to measure the longitudinal, lateral and vertical acceleration, the roll, yaw and pitch rate, the steering angle and the lean angle (the lean angle only for the three wheels scooter). Both the gyroscopes and the accelerometers were installed in the sensor box A located in the rear part of the vehicle. (Figure 6a). The wire potentiometer, evident in (Fig. 6b), was installed to measure the steering angle. A DSP based acquisition board (B in Figure 6a) was used to acquire all the sensor signals.

The tests were carried out exciting the steering axis with an impulsive torque while straight running at different speeds. This leads to excite the weave and wobble modes. The impulsive force was applied by the driver hitting one side of the handlebar with one hand (without holding the handlebar).

The comparison of the numerical and experimental steering angle during the transient after excitation at 65 km/h (Fig. 8a and 9a) allows identifying the steering damping. This identification has been performed for the Matlab/SimMechanics model described in the previous sections and, in the case of the two wheels scooter, for an Adams Motorcycle model.

The response of the models with the identified values is then compared to the experimental ones at higher speeds showing a good correlation (Fig. 7b and 8b). This is considered as a proof of the validity of the models. The results reported in Fig. 7 and 8 evidence the higher stability of the three wheels with respect to the two wheels scooter.
The development of the Adams three wheels vehicle requires the implementation of a non-conventional front body structure and is currently in progress.

**Figure 7**: Comparison between the numerical and experimental time history of the steering angle. The data are referred to the two wheels scooter.

![Figure 7](image_url)

**Figure 8**: Comparison between the numerical and experimental time history of the steering angle. The data are referred to the three wheels scooter.

![Figure 8](image_url)

**Stability analysis**

The analysis of the stability is investigated in the present section using the numerical models previously validated. To this end, the eigenvalue of each mode is computed in the speed range of interest (20-240 km/h) considering the vehicle at constant speed during straight running. The results are plotted in the root loci graph of Fig. 10. In the same graph are reported the root loci (circle marks) extrapolated by the experimental curves of Fig. 8 and 9. The good correlation between the numerical and experimental results is confirmed also by these graphs.

The comparison of the root loci evidences the following considerations:

- the weave mode of both the vehicles moves towards the threshold of instability with increasing velocities. The frequency of the three wheels vehicle remains quite constant while that of the two wheels one increases with the speed. Additionally, the weave mode of the two wheels scooter becomes unstable at a speed that is about 20 km/h lower than the three wheels one
- The wobble modes of the three wheels scooter results to be much damped and therefore far from becoming unstable. On the contrary, most of the roots related to the two wheels vehicle lies on the imaginary axis for a wide speed range
It results that the three wheels scooter is more stable than the two wheels one for the speed range of interest. This is ascribed to

- the higher damping introduced by the friction of the steering system joints. It has been identified to be three times higher than the friction introduced by the bearings supporting the front body of the two wheels vehicle
- The higher stiffness and structural damping of the three wheels scooter chassis. The wobble modes are very sensitive to those parameters and a chassis stiffness and damping, that is only the 20 % higher in the three wheels scooter, is effective on the wobble modes behaviour

It must be added that the three wheels scooter has a lower front wheel castor trail and a lower front body inclination. This was needed to reduce the steering torque at zero speed for parking manoeuvres. The effect of reducing the castor trail and the front body inclination is that of reducing the damping of the wobble modes. This means that the effect of the damping in the steering joints and of the chassis torsional stiffness and damping is mitigated in the present analysis by the reduction of the castor trail and the front body inclination. Considering the same castor trail and front body inclination, the effect of the steering damping and of the chassis stiffness and damping would result much more evident.

![Figure 9: Root loci of the two wheels (squared markers) and three wheels (star markers) scooters. Min speed=20 km/h, max speed 240 km/h, straight running.](image)

**Conclusions**

The present work introduces the multi-body model of a three tilting wheels maxiscooter including the effects evidenced in most recent literature on motorcycles, such as the tyre relaxation length and the chassis compliance.

The model has been validated by the comparison of the numerical and experimental time history of the steering angle during straight running at different speeds. The weave and wobble modes are excited by hitting the handle bar without any vehicle control.

The stability analysis is performed on the validated models by means of a root loci plot as function of the vehicles speed. The eigenvalues of each mode are compared to those of a conventional maxiscooter having similar characteristics.

The stability analysis underlines that:

- the main dynamics (weave and wobble) of non conventional three wheels tilting vehicles are similar to the ones of a standard motor bike.
- The three wheels vehicle studied in the present paper is much more stable than the equivalent two wheels maxiscooter. This is ascribed to the higher damping introduced by the steering system and by the higher stiffness and damping of the main frame.
The model described and validated in the present paper can be considered a valid tool for the definition of some design parameters such as the castor trail and the body stiffness.

References


